Supplementary Material for FactoredSweeper: Optical See-Through Display Integrating Light Attenuation and Addition with Single Spatial Light Modulator

1 Color Differences for Objective Function

1.1 Objective Function with CIE94

CIE94 ΔE_{94} is designed to provide metrics closer to human color perception than CIE76. ΔE_{94} is calculated as

$$\Delta E_{94}(\mathbf{x}^1_{Lab},\mathbf{x}^2_{Lab}) = \sqrt{\left(\frac{\Delta L^*}{k_L S_L}\right)^2 + \left(\frac{\Delta C^*}{k_C S_C}\right)^2 + \left(\frac{\Delta H^*}{k_H S_H}\right)^2}, \quad (14)$$

where

$$C^* = \sqrt{(a^*)^2+(b^*)^2}, \quad \Delta L^* = L^*_1 - L^*_2, \quad \Delta C^* = C^*_1 - C^*_2,$$
$$\Delta a^* = a^*_1 - a^*_2, \quad \Delta b^* = b^*_1 - b^*_2,$$
$$\Delta H^* = \sqrt{(\Delta a^*)^2 + (\Delta b^*)^2 - (\Delta C^*)^2},$$
$$S_L = 1, \quad S_C = 1 + K_1 C^*_1, \quad S_H = 1 + K_2 C^*_1. \quad (15)$$

In addition, k_L, k_C and k_H are adjustment factors for each environmental condition and are normally set to 1 under standard conditions. K_1 and K_2 are weighting factors set to $K_1 = 0.045$ and $K_2 = 0.015$ for image evaluation.

With ΔE_{94}, we rewrite Eq. (9) as

$$\underset{\{\Gamma,\mathbf{H}\}}{\text{minimize}} \quad \|\Delta E_{94}(\beta \mathbf{I}_{Lab}, \varphi(\mathbf{X}(\Gamma,\mathbf{H})))\|^2_F, \quad (16)$$
$$\text{s.t.} \quad 0 \le h_{m,n} \le 1;\ \forall m,n.$$

1.2 Objective Function with CIEDE2000

CIEDE2000 ΔE_{00} is calculated as

$$\Delta E_{00} = \sqrt{\left(\frac{\Delta L^*}{k_L S_L}\right)^2 + \left(\frac{\Delta C'}{k_C S_C}\right)^2 + \left(\frac{\Delta H'}{k_H S_H}\right)^2 + \frac{R_T \Delta C' \Delta H'}{k_C S_C k_H S_H}}$$

where

$$\bar{L} = \frac{L^*_1 + L^*_2}{2}, \quad \bar{C} = \frac{C^*_1 + C^*_2}{2},$$

$$a' = a^* + \frac{a^*}{2}\left(1 - \sqrt{\frac{\bar{C}^7}{\bar{C}^7 + 25^7}}\right), \quad C' = \sqrt{(a')^2 + (b^*)^2},$$

$$\bar{C'} = \frac{C'_1 + C'_2}{2}, \quad \Delta C' = C'_2 - C'_1, \quad h' = \text{atan2}(b^*, a') \bmod 360^\circ,$$

$$\Delta h' = \begin{cases} h'_2 - h'_1 & |h'_1 - h'_2| \le 180^\circ \\ h'_2 - h'_1 + 360^\circ & |h'_1 - h'_2| > 180^\circ,\ h'_2 \le h'_1 \\ h'_2 - h'_1 - 360^\circ & |h'_1 - h'_2| > 180^\circ,\ h'_2 < h'_1 \end{cases}$$

$$\Delta H' = 2\sqrt{C'_1 C'_2}\sin\left(\frac{\Delta h'}{2}\right),$$

$$\bar{H}' = \begin{cases} (h'_1 + h'_2)/2 & |h'_1 - h'_2| \le 180^\circ, \\ (h'_1 + h'_2 + 360^\circ)/2 & |h'_1 - h'_2| > 180^\circ,\ h'_1 + h'_2 < 360^\circ, \\ (h'_1 + h'_2 - 360^\circ)/2 & |h'_1 - h'_2| > 180^\circ,\ h'_1 + h'_2 \ge 360^\circ, \end{cases}$$

$$S_L = 1 + \frac{K_1(\bar{L} - 50)^2}{\sqrt{20 + (\bar{L} - 50)^2}}, \quad S_C = 1 + K_2\bar{C}, \quad S_H = 1 + K_1\bar{C}T,$$

$$\begin{aligned} T &= 1 - 0.17\cos(\bar{H}' - 30^\circ) + 0.24\cos(2\bar{H}') \\ &\quad + 0.32\cos(3\bar{H}' + 6^\circ) - 0.20\cos(4\bar{H}' - 63^\circ), \end{aligned}$$

$$R_T = -2\sqrt{\frac{\bar{C}^7}{\bar{C}^7 + 25^7}}\sin\left[60^\circ \exp\left(-\left[\frac{\bar{H} - 275^\circ}{25^\circ}\right]^2\right)\right]. \quad (17)$$

2 Optimization of Comparison Methods

We describe the optimization details of the comparison method used in the simulation and the actual measurements. We denote the subsets of Ψ_d as $\Psi^{\mathrm{S}}_d = [\Psi_{d,[:,\,M+1:M]}] \in \mathbb{R}^{L\times M} (d = 1,2)$, the subset of Γ as $\Gamma_M = \Gamma_{[1:M]}$, and $\mathbf{G}_M = \text{diag}(\mathbf{g}(\Gamma_M))$.

In the absence of color filters, we instead introduce a weight α_w for the real light contribution. To align the conditions of our method with those of other methods for the contribution of light to reality, we define this α_w as the brightest white that can be represented by the weighted sum of the color filters when the spectrum of reality passes through a color filter. The formulation can be written as

$$\mathbf{i}^w_{XYZ} = [X^w, Y^w, Z^w]^{\mathrm{T}} = \mathbf{P}_{XYZ}\Psi^{\mathrm{S}}_1 \mathbf{g}(\Gamma_M)). \quad (18)$$

To make this $\mathbf{i}^w_{XYZ}$ the "brightest white", we optimize Γ_M to satisfy $X^w = Y^w = Z^w$ and maximize Y^w as

$$\underset{\Gamma_M}{\text{minimize}}(|X^w - Y^w| + |X^w - Z^w| + |Y^w - Z^w| - Y^w), \quad (19)$$

then we set $\alpha_w = Y^w/100$. Using AdamW optimizer with lr=1e-2 and 1000 iterations, we found $\alpha_w = 0.178$ in our setup.

2.1 Addition-only OST-HMD

In the case of additive OST-HMDs, where the virtual image is overlaid on the globally dimmed environmental light (e.g., Microsoft Hololens 2), the real environment can be considered to be uniformly dimmed and then light is added. As with other methods, we assume that an optical path using LEDs and color filters is used to form the virtual image. The image formation can then be formulated as

$$i(\lambda,x) \quad = \quad \alpha_w r(\lambda,x) + \sum_{m=1}^{M} g_m \cdot l(\lambda)\psi_m(\lambda)h_m(x).$$

Discretizing this equation, we obtain

$$i_{l,n} \quad = \quad \alpha_w r_{l,n} + l_l \sum_{m=1}^{M} \psi_{l,m} g_m h_{m,n}.$$

With Ψ_S, this image formation is obtained as matrix form:

$$\mathbf{I} \quad = \quad \alpha_w \mathbf{R} + \Psi^{\mathrm{S}}_2 \mathbf{G}_M \mathbf{H}^{\mathrm{T}}. \quad (20)$$

2.2 Single-SLM Light Attenuation Display

In the initial single-SLM LAD [17], the optimization is performed only on CIEXYZ, and only 25 colors in the color chart are optimized at most. Therefore, to extend this image formation formula to PNMF and to represent any color, we formulated the subtruct-only image as

$$i(\lambda,x) \quad = \quad \sum_{m=1}^{M} g_m \cdot r(\lambda,x)\psi_m(\lambda)h_m(x).$$

Discretizing this equation, we obtain

$$i_{l,n} \quad = \quad r_{l,n}\sum_{m=1}^{M} \psi_{l,m} g_m h_{m,n}.$$

This image formation is obtained as matrix form:

$$\mathbf{I} \quad = \quad \mathbf{R} \odot (\Psi^{\mathrm{S}}_1 \mathbf{G}_M \mathbf{H}^{\mathrm{T}}). \quad (21)$$

2.3 Single-SLM OCOST-HMDs

In Fig. 4, the single-SLM OCOST-HMD can be considered to have a color filter $\psi_{M+1}(\lambda) = 0$ when the LED is off. At this time, the color filters contribute only to the formation of the virtual image. The overall image formation can be formulated as

$$\begin{aligned} i(\lambda, x) &= \sum_{m=1}^{M+1} g_m \cdot \alpha_w r(\lambda, x) h_m(x) \\ &+ \sum_{m=1}^{M+1} g_m \cdot l(\lambda) \psi_m(\lambda)(1 - h_m(x)). \end{aligned}$$

Discretizing this equation, we obtain

$$i_{l,n} = \alpha_w r_{l,n} \sum_{m=1}^{M+1} g_m h_{m,n} + l_l \sum_{m=1}^{M+1} \psi_{l,m} g_m (1 - h_{m,n}).$$

This image formation is obtained as matrix form:

$$\begin{aligned} \mathbf{I} &= \alpha_w \mathbf{R} \odot (\mathbf{1}_{L \times (M+1)} \mathbf{G}_{M+1} \mathbf{H}^{\mathrm{T}}) \\ &+ [\Psi_2^{\mathrm{S}} \mid \mathbf{0}_{M+1}] \mathbf{G}_{M+1} (\mathbf{1}_{N \times (M+1)} - \mathbf{H})^{\mathrm{T}}. \end{aligned} \quad (22)$$